Escuela de Invierno

Tutorial I: Tabu Search

Jornadas de Optimización Heurística

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21-23 noviembre 2018

Tabu Search

Fred Glover 1986

Fred with our little Tabu Search

History

- A very simple memory mechanism is described in Glover (1977) to implement the *oscillating assignment* heuristic.
- Glover (1986) introduces *tabu search* as a "meta-heuristic" superimposed on another heuristic

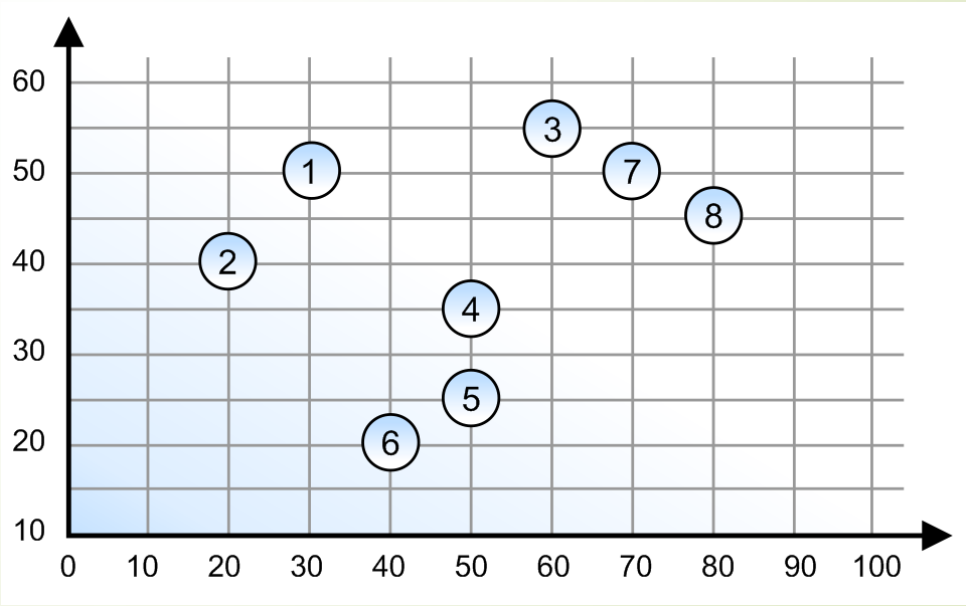
- Glover, F. (1977) "Heuristics for Integer Programming Using Surrogate Constraints," *Decision Sciences*, vol. 8, no. 1, pp. 156-166.
- Future paths for integer programming and links to artificial intelligence, Computers & Operations Research 13(5), 533-549, 1986.
- Glover, F. (1989a) "Tabu Search – Part I," INFORMS J. on Comp. 1(3), 190-206.
- Glover, F. (1989b) "Tabu Search – Part II," INFORMS J. on Comp. 2(1) 4-32.

An Example

The maximum diversity problem

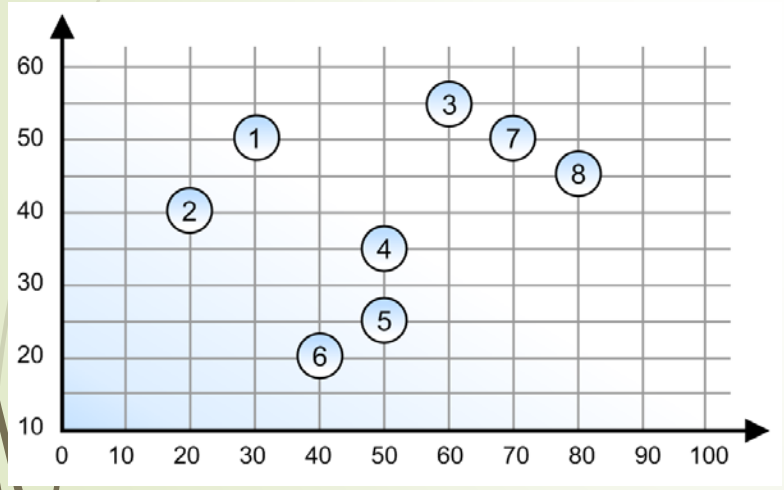
- Consists of selecting a subset of *m* elements from a set of *n* elements to maximize the sum of the distances between the chosen elements.

	Age	Incomes
1	50	30
2	40	20
3	55	60
4	35	50
5	25	50
6	20	40
7	50	70
8	45	80



Create a comitee with 4 employees

- The distance between points in the graphic reflects the difference between the associated employees
- In this example, we use the **Euclidean distance** to calculate the distance between the elements, but other metrics can be used as well.



	2	3	4	5	6	7	8
1	14	30	25	32	32	40	50
2		43	30	34	28	51	60
3			22	32	40	11	22
4				10	18	25	32
5					11	32	36
6						42	47
7							11

Distance Matrix

Diversity Evaluation

- The diversity of the chosen elements is the **sum** of the distances between them

	2	3	4	5	6	7	8
1	14	30	25	32	32	40	50
2		43	30	34	28	51	60
3			22	32	40	11	22
4				10	18	25	32
5					11	32	36
6						42	47
7							11

Selected employees

$x = \{ 3, 4, 6, 8 \}$

22 40 22 18 32 47

sum

$z(x) = 181$

The **MaxSum** model resolution consists of finding the set of 4 employees with the highest **sum** of distances

Adaptive Memory Programming

- The simpler Tabu Search (TS) method incorporates a restricted portion of the TS design, usually involving only short term memory.
- The more advanced method embodies a broader framework that includes longer term memory, with associated intensification and diversification strategies.
- This second approach, due to its focus on exploiting a collection of strategic memory components, is sometimes referred to as *Adaptive Memory Programming* (AMP). It also includes constructive and destructive neighborhoods.

Simple Tabu Search Design

- Construct an Initial Solution
- Repeat until some termination criterion is satisfied:
  - Change something in the current solution (but do not change anything that is **tabu-active**).
  - Declare what you just changed tabu-active.
- Tabu items are only **remembered** for a short period of time.

Memory Types

- Explicit Memory
  - A record of complete solutions
  - Avoiding visiting solutions more than once (limited use).
  - Elite solutions in tabu search may be used to expand local search.
- Attributive Memory
  - A record of attributes (properties) that change when moving from one solution to another.

First Improving Local Search

- Choose the first improving move during the exploration of the current neighborhood
- This is a special case of the *Aspiration Plus* Candidate List Strategy
  - Threshold = Current Solution Value
  - Plus = 0
  - Min = 0
  - Max = Size of the neighborhood

Tabu Local Search

Recency-Based Memory

- Short term memory most commonly used in tabu search implementations.
- It keeps track of solution **attributes** that have changed during the recent past.
- Selected attributes that occur in solutions recently visited are labeled **tabu-active**.
- Solutions that contain tabu-active elements, or particular combinations of these attributes, are those that become **tabu**.

MDP: Short Term Memory

- The **short-term tabu search** method is also based on exchanges.
- The probability of selecting element i is inversely proportional to D(i). The list of unselected elements is scanned and the first improving move that exchanges elements is selected.
- If no improving move is found, then **the least non-improving move is chosen**, and both elements participating in the exchange are classified tabu-active (with an asymmetric tabu tenure).

Initial Solution

- Random, or in general without any problem-knowledge.
- Based on a systematic process:
  - Constructive method
- Based on a Multi-start method.
  - Iterative constructive method

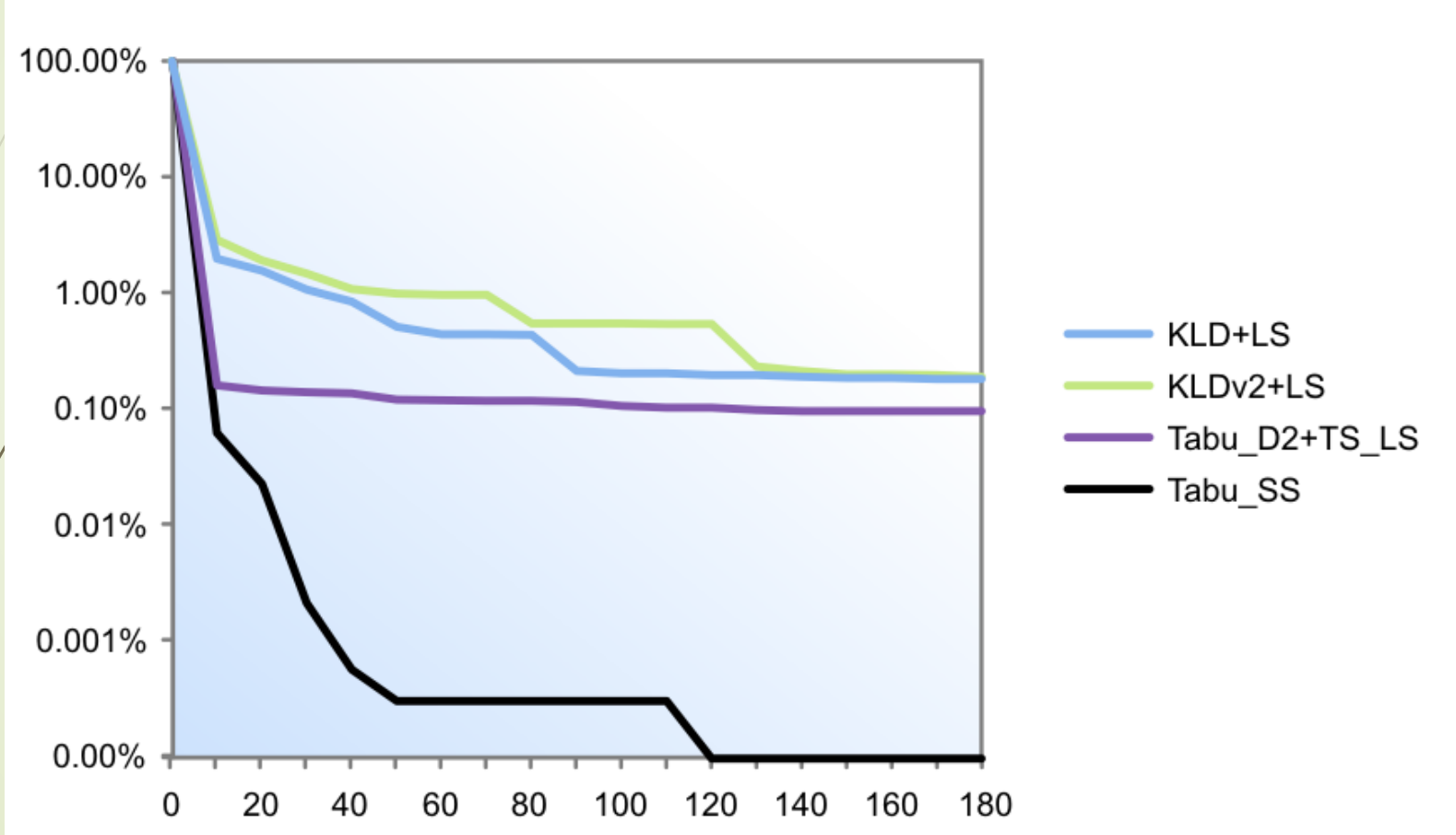
MDP: Tabu Destructive Method

- Apply memory to a deterministic destructive heuristic.
- Consider that all the elements are selected.
- Greedy destructive method: Des-select the one with lowest contribution (D(i)=Sum of distances to selected elements) at each step.
- Modify the greedy value D(i) with a frequency and a quality .
- Compute the At each step, the procedure deselects element i* such that:

$$D(i^*) = \min_{i:x_i=1}\left(D(i) - \beta(range)\left(\frac{f(i)}{f_{\max}}\right) + \delta(range)\left(\frac{q(i)}{q_{\max}}\right)\right)$$

$$range = \max_{i:x_i=1}(D(i)) - \min_{i:x_i=1}(D(i))$$

Tabu Search Profile on MDP



Long Term Memory

- Frequency-based memory
- Strategic oscillation
- Path relinking

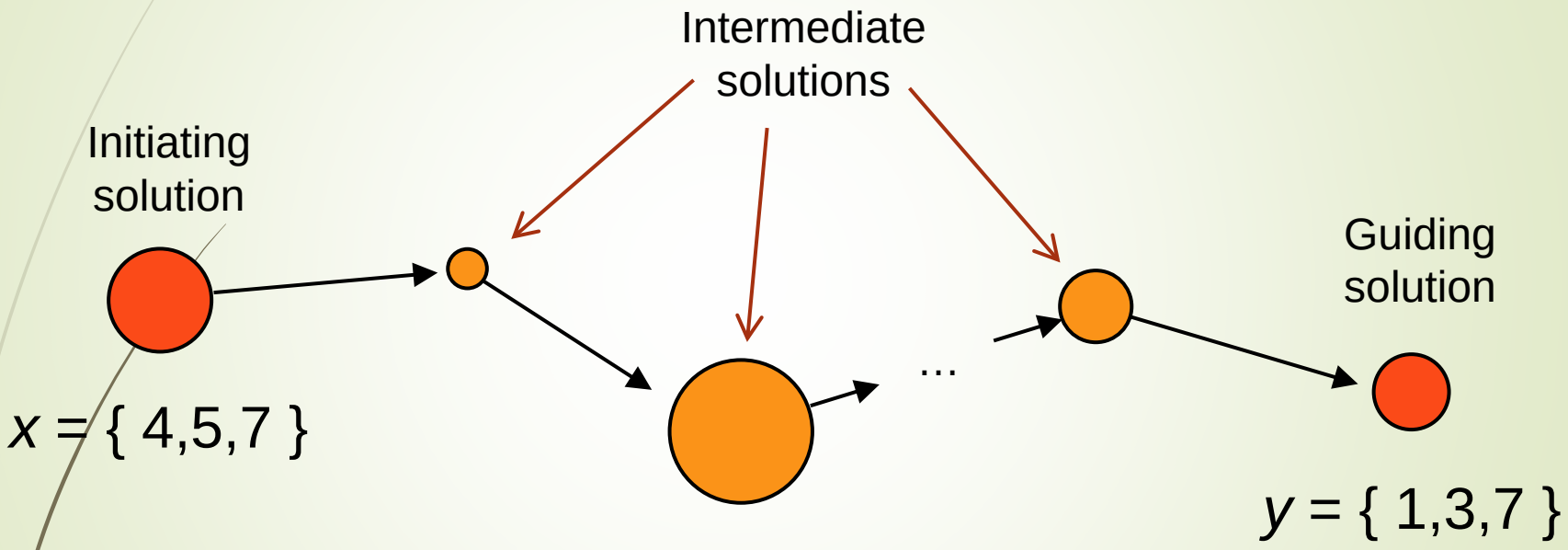
Frequency-based Memory

- Transition Measure
  - Number of iterations where an attribute has been changed (e.g., added or deleted from a solution)
- Residence Measure
  - Number of iterations where an attribute has stayed in a particular position (e.g., belonging to the current solution)

Strategic Oscillation

- Strategic oscillation operates by orienting moves in relation to a boundary
- Such an oscillation boundary often represents a point where the method would normally stop or turn around

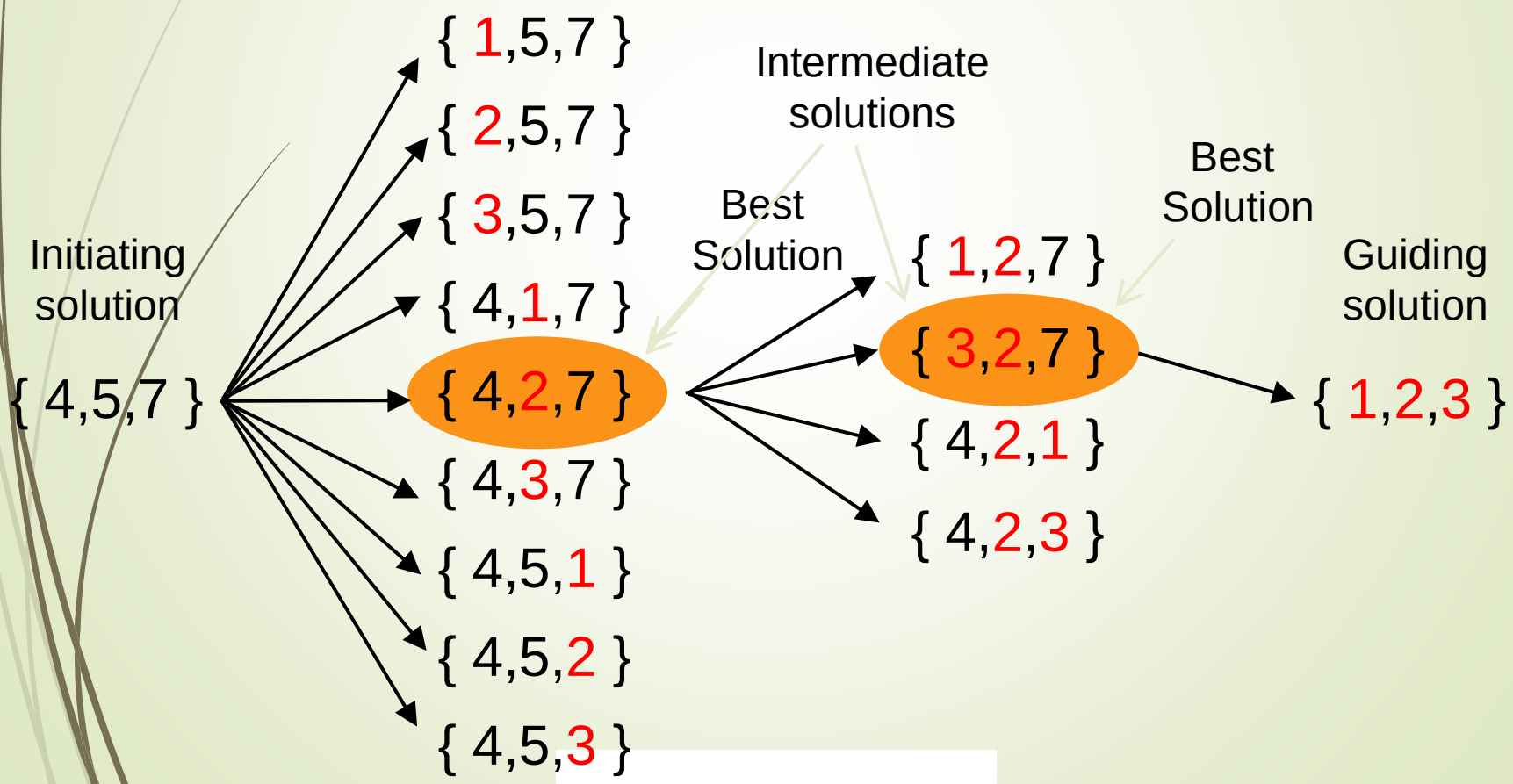
Post-Processing in Tabu Search:
Path Relinking



- In the path between two high-quality solutions new quality solutions can be found

Greedy Path Relinking (GPR)

- Select best solution in each step



Greedy Randomized PR (GPR)

- Select randomly from the best solutions in each step

Relinking Strategies

- *Periodical relinking* — not systematically applied to all solutions
- *Forward relinking* — worst solution is the initiating solution
- *Backward relinking* — best solution is the initiating solution
- *Backward and forward relinking* — both directions are explored
- *Mixed relinking* — relinking starts at both ends
- *Randomized relinking* — stochastic selection of moves
- *Truncated relinking* — the guiding solution is not reached

Related TS Methods

- Probabilistic Tabu Search
- Tabu Thresholding
- Reactive Tabu Search

Conclusions

- Memory seems to be a good idea
- A low level of randomization is also a good idea

- Combining randomization with memory leads to advanced designs.

  Is that the foundation of human thinking?

Recommendations

Lessons from heuristic research

- Don’t base your heuristic in all the methodologies!
- Design your initial method following the specifications of a well-established methodology
  - *Just follow the rules*
- Introduce changes, new elements, and strategies to create and advanced design
  - *Propose now what you want*
- You can even hybridize the method: add elements taken from another methodology
  - *Please, just one additional methodology*
- Compare your advanced design with the initial one
  - Perform scientific comparisons of your elements
  - *If no improvement is observed, goto Day 1*